Physics 402

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1. The electron in a hydrogen atom occupies the combined spin and position state

$$\Psi = R_{21}(r)\left(\sqrt{\frac{1}{3}}\, Y_1^0(\theta,\phi)\,\chi_+ + \sqrt{\frac{2}{3}}\, Y_1^1(\theta,\phi)\,\chi_-\right)$$

a) If you measured the orbital angular momentum squared (L^2), what values might you get, and what is the probability of each?
b) Same for the z component of orbital angular momentum (L_z)
c) Same for the spin angular momentum squared (S^2)
d) Same for the z component of spin angular momentum (S_z)

a) L^2 operates on the Y_ℓ^m and returns eigenvalue $\ell(\ell+1)\hbar^2$. Both spherical harmonics have $\ell=1$. Taking the expectation value of L^2 in this state will return

$1(1+1)\hbar^2$ with prob. $1/3$ from the first term
$1(1+1)\hbar^2$ " " $2/3$ " " second "

Hence $\langle L^2\rangle = 2\hbar^2$ with probability 1.

b) L_z only operates on the Y_ℓ^m and returns eigenvalue $m\hbar$. This yields

$0\hbar$ with prob. $1/3$
$1\hbar$ with prob. $2/3$

c) S^2 only operates on the spin states $\chi_\pm$. It returns eigenvalue $s(s+1)\hbar^2$, where $s=1/2$ for the electron.

$\frac{1}{2}(\frac{1}{2}+1)\hbar^2$ with prob. $1/3$ (first term)
$\frac{1}{2}(\frac{1}{2}+1)\hbar^2$ with prob $2/3$ (2^{nd} term)

Hence the $\langle S^2\rangle = \frac{3}{4}\hbar^2$ with probability 1.

d) S_z only operates on the spin states $\chi_\pm$, returning eigenvalues $\pm\frac{\hbar}{2}$.

$+\frac{\hbar}{2}$ with prob. $1/3$
$-\frac{\hbar}{2}$ with prob. $2/3$